Are Stock Prices Hedge Against Inflation? A Revisit over Time and Frequencies in India

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Abstract

In this paper, the stock price-inflation nexus is investigated using the tools of wavelet power spectrum, cross-wavelet power spectrum and cross-wavelet coherency to unravel time and frequency dependent relationships between stock prices and inflation. Our results suggest that for a frequency band between sixteen and thirty two months, there is some evidence of the fisher effect. For rest of the frequencies and time periods however there is no evidence of the fisher effect and it seems stock prices have not played any role as an inflation hedge.

Keywords: Stock prices, inflation, Fisher effect, Indian stock markets, continuous wavelet transform, wavelet coherency

JEL Classification: C40, G10, G12, E31

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1 Introduction

The Fisher hypothesis in its most common version asserts for a one for one movement between the expected nominal asset returns and the expected inflation. This implies that stock market serves as a hedge against inflation; that is, investors are compensated fully for the increased general price level through corresponding increase in the nominal stock returns. This issue has been analysed extensively in the literature and a number of studies have documented the positive correlation between nominal stock return and inflation rate (see for example, Firth, 1979; Gultekin, 1983; Kessel, 1956; Ioannides, Katrakilidis, Lake 2005). For the real stock return, the conventional Fisherian wisdom argues for a positive or at least a non-negative relation with inflation rate (Durai and Bhaduri, 2009). Several studies, however, have discovered significant negative correlation between the real stock and inflation, rejecting the Fisher hypothesis (Fama and Schwert, 1977; Barnes, Boyd, Smith 1999; Spyrou, 2001). Defending the observed negative association, Fama (1981) has forwarded two propositions that links real stock return and inflation through real output. First, there is a negative relationship between inflation and real output. Second there is a positive relationship between real output and real stock return. More specifically, since stock prices reflect firms’ potentiality for future earning, an economic downturn predicted by a rise in inflation is likely to depress stock prices.

A vast body of literature centred on the empirical relevance of the hedge hypothesis is highly biased and lop-sided, focusing mostly over short horizons; typically less than a year. Nevertheless, analysis over longer horizon is crucial for having greater insights into the issue. The peculiarity of financial market is underlined with the existence of highly heterogeneous market participants who do trading at different frequencies. Among the different frequency traders, institutional investors constitute low frequency traders whereas speculators and market makers come under the category of high frequency traders. These market participants differ on the basis of their beliefs, expectations, risk profiles as well as informational sets. The presence of market heterogeneity, therefore, warrants the analysis of the stock price-inflation relationship at different frequencies.

The earlier Studies like Solnik and Solnik (1997), Engsted and Tanggaard (2002), and Schotman and Schweitzer (2000) studied the relation of stock returns and inflation over long horizons and observed increasing evidence of the Fisher hypothesis as the time horizon increases. Moreover, recent strand of literature e.g. Kim and In (2005), Durai and Bhaduri (2009) study this relationship focusing on different frequencies by decomposing the inflation and stock prices with the discrete wavelet transforms. It must be noted, however, that the use of discrete wavelet transform invokes arbitrariness in the choice of number of scales, which may render the results meaningless. However, the application of wavelet analysis in their study is limited to the use of MODWT for decomposition and subsequent use of econometric approach to study the relationship. The relationship in time series data, what we may get by utilizing any method of discrete wavelet transformation at each scale, can be
obtained more easily with continuous transformation without relying on econometric techniques. In particular, through continuous wavelets one can assess simultaneously the strength of the co-movement at different frequencies and over time. One can also identify regions in the time-frequency space where co-movement is high and regions where co-movement is low. In other words, this methodology helps to unravel local correlation and local causal relationship over different time periods and frequencies without relying on traditional econometric techniques.

On this backdrop, this study proposes a new approach, the continuous wavelet analysis, for investigating the relationship between nominal stock returns and inflation across different periods and over different frequencies. Following Kim and In (2005), we further analyse whether the expected nominal stock returns, compared to the real returns, are related differently to the inflation over different time horizons. The main results from the empirical analysis indicate that for the entire sample period, stock returns have not played any significant role as inflation hedge, except for the short period of 1991-1993 over 16 ~ 32 months time scale. This argument is, however, based on the premise that, only positive relation (Fisher effect) does not hold.

The paper is organised as follows. Section 2 sets out the data, motivation along with the methodology applied for the empirical analysis. The results and discussions are presented in Section 3. Section 4 finally, summarizes and concludes.

2 Data and methodology

2.1 Data

For the empirical estimation, monthly data spanning from May 1960 through January 2009 for share prices and consumer price index (CPI) were utilised. The data span ensures that we have sufficient observations and reliable results. The share prices and CPI variable were obtained from the International Monetary Fund (IMF) CD-ROM of International Financial Statistics (IFS), 2010. We transformed both variables into their log first difference to have share returns (proxy for stock returns) and inflation.

2.2 Motivation and introduction to methodology

Until recent past, the stock price-inflation relationship has been studied in conventional time domain framework ignoring fluctuations over frequencies. The dynamic relationship between stock prices and inflation may exhibit appealing features, which can also vary across different frequencies. According to heterogeneous market hypothesis, the traders can be classified into their characteristic time horizons or dealing frequencies. Among the different frequency traders, institutional investors constitute low frequency traders whereas, speculators and market-makers are categorised under high frequency traders. The presence of market heterogeneity, therefore, leads to presence of different dealing frequencies, and thus different reactions...
to the same news in the same market. Each market component has its own reaction
time to information, related to its time horizon and characteristic dealing frequency
(Dacorogna, Gençay, Müller, Olsen, Pictet 2001). Thus, true economic relationships
among variables can be expected to hold at the disaggregated (scale) level rather
than at the usual aggregate level. Some studies like Kim and In (2005), Durai and
Bhaduri (2009) have used the MODWT to establish multi-scale relationship between
stock prices and inflation. However, continuous wavelet transform can confer several
benefits over MODWT (see introduction section for details).

Using continuous wavelet transform, Torrence and Compo (1998) developed
approaches of the cross-wavelet power, the cross-wavelet coherency, and the phase
difference. These analytical tools allow us to directly study the interactions between
any two time series at different frequencies and over time. Wavelet power spectrum
is suitable to understand the evolution of the variance of a time series at the different
frequencies, with periods of large variance associated with periods of large power at
the different scales. The cross-wavelet power of two time series defines the covariance
between the time series and the wavelet coherency can be interpreted as correlation
coefficient in the time-frequency space. In strict sense however, the wavelet coherency
is not synonymous to correlation, because it measures correlation at all leads and
lags simultaneously. We are thankful to reviewers for highlighting this issue. Thus,
coherence is bounded between 0 and 1 while coherency is bounded between -1 and 1.
The term 'phase' implies the position in the pseudo-cycle of the series as a function
of frequency. Consequently, the phase difference gives us information 'on the delay,
or synchronization, between oscillations of the two time series' (Aguiar-Conraria,

The description of CWT, XWT and WTC is heavily drawn from Grinsted et al.
(2004). We are grateful to Grinsted and co-authors for making codes available at:
http://www.pol.ac.uk/home/research/waveletcoherence/, which was utilized in
the present study.

2.2.1 The Continuous Wavelet Transform (CWT)

The objective of the wavelet analysis is to determine the frequency content of a
variable with a view to extracting the temporal variation of this frequency content
(Heil and Walnut, 1989; Labat, 2005). A wavelet is a function with zero mean
localized in both time and frequency. It grows quickly and decays within a limited
period (Fan and Gancéy, 2010) thereby obeying the conditions that \( \int \psi(\eta) d\eta = 0 \)
and \( \int |\psi(\eta)|^2 d\eta = 1 \). We can characterize a wavelet by its localization in time \((\Delta t)\) and
frequency \((\Delta \omega \text{ or the bandwidth})\).

Thus, for a CWT of series \( x(t) \)

\[
W_x(s, \tau) = \langle x(t), \psi(t) \rangle = \int_{-\infty}^{\infty} x(t) \ast \frac{1}{\sqrt{s}} \psi \left( \frac{t - \tau}{s} \right) dt
\]

(1)
where $s$ and $\tau$ are the scale and location parameters and $\psi \left( \frac{t-\tau}{s} \right)$ is known as the mother wavelet function that is possibly complex-valued. The symbol $\circ$ is the convolution operator. A complex wavelet function is of valuable utility in economic analysis as it gives information on local phase. One such function having this property is the Morlet wavelet function. Besides, the Morlet wavelet function can be shown to achieve an optimal localization between the resolution in time and in frequency due to its Gaussian envelop. This property is guaranteed by Heisenberg’s uncertainty theorem stating that there is a lower limit to the product of time and frequency resolution. Also implying a tradeoff between the resolution in time and in frequency, the theorem ensures that any improvement in time degrades the frequency resolution and any improvement in frequency degrades the time resolution. Thus, to achieve optimal balance, we employ the Morlet wavelet function given by

$$
\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\frac{1}{2} \eta^2}
$$

where $\omega_0$ is dimensionless frequency and $\eta$ is dimensionless time. For optimal balance, we set $\omega_0 = 6$ as suggested by Torrence and Compo (1998). Since the idea behind the CWT is to apply the wavelet as a band pass filter to the time series, the wavelet is stretched in time by varying its scale $s$, so that $\eta = s \cdot t$ and normalizing it to have unit energy. For the Morlet wavelet, the Fourier period ($\lambda_{wt}$) is almost equal to the scale ($\lambda_{wt} = 1.03$). The wavelet transform also inherits this property.

The discretized version of Equation (1) for time series $\{x_n : n = 1, \ldots, N\}$ is given by

$$
W^x_{m}(s) = \frac{\delta t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \cdot \psi^* \left( (m-n) \frac{\delta t}{s} \right), \ m = 1, 2, \ldots, N - 1
$$

where $\delta t$ is the uniform step size. From the expression above, the wavelet power that measures the variability in the time series both in time and in frequency is defined as $|W^x_{m}(s)|^2$. For this discretized version, the complex argument of $W^x_{m}(s)$ can be interpreted as the local phase. Specifically, if $W^x_{m}(s)$ is complex-valued, then it can be separated into real $\Re\{W^x_{m}(s)\}$ and imaginary $\Im\{W^x_{m}(s)\}$ parts, allowing for the calculation of the phase angle, $\varpi_x = \tan^{-1} (\Im\{W^x_{m}(s)\}/\Re\{W^x_{m}(s)\})$ parameterized in radians ranging from $-\pi$ to $\pi$. The CWT suffers from edge effects caused by a discontinuity at the edge because wavelet is not completely localized in time. To cope with this challenge, the cone of influence (COI) has been introduced. The COI earmarks the area where edge effects cannot be ignored and determines the set of CWT coefficients influenced by the value of the signal at a specified position. Outside COI, edge effects are predominant and can distort the result. Here we take the COI as the area in which the wavelet power drops to $e^{-2}$ of the value at the edge.

### 2.2.2 Wavelet Coherence (WTC)

Since our intention is to measure the extent of synchronization between two given time series, it is informative to use coherence between them. Wavelet coherence is a
time-frequency counterpart of the time-domain coefficient of determination and shares property with traditional correlation coefficient. Aguiar-Conraria, Azevedo, Soares, (2008) defines wavelet coherence as “the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local (both in time and frequency) correlation between two time-series”. Following Torrence and Webster (1999), we define the wavelet coherence between two time series as

$$R_m(s) = \frac{|S \left( s^{-1}W_{xy}^m(s) \right)|}{S \left( s^{-1}|W_{xm}^m(s)|^{\frac{1}{2}} \right) \cdot S \left( s^{-1}|W_{ym}^m(s)|^{\frac{1}{2}} \right)},$$

where $S$ is a smoothing operator and $W_{xy}^m = E \left( W_x^m \overline{W_y^m} \right)$ is the cross-spectrum, with $\overline{W_y^m}$ as the complex conjugate of $W_y^m$. Notice that $0 \leq R_m(s) \leq 1$ while for the traditional correlation coefficient ($\rho$) $0 \leq \rho \leq 1$. Without smoothing coherency is identically 1 at all scales and times. We may further write the smoothing operator $S$ as a convolution in time and scale:

$$S(W) = S_{scale} \left( S_{time} (W_m(s)) \right)$$

where $S_{scale}$ denotes smoothing along the wavelet scale axis and $S_{time}$ denotes smoothing in time. The time convolution is done with a Gaussian and the scale convolution is performed with a rectangular window (see, for more details, Torrence and Compo 1998).

It is important to conceptualize the lead-lag relationship between two time series. This is achieved by computing the phase difference given by

$$\phi_{x,y} = \tan^{-1} \frac{\Im \{ W_{xy}^m \}}{\Re \{ W_{xy}^m \}}, \quad \phi_{x,y} \in [-\pi, \pi]$$

where $\Im$ and $\Re$ are the imaginary and real parts of the smooth power spectrum respectively. Phase differences are useful to characterize phase relationship between any two time series. A phase difference of zero indicates that the time series move together at the specified frequency. If $\phi_{x,y} \in [0, \frac{\pi}{2}]$, then the series move in-phase, with the time-series $y$ leading $x$. On the other hand, if $\phi_{x,y} \in [-\frac{\pi}{2}, 0]$ then $x$ is leading. We have an anti-phase relation (analogous to negative covariance) if we have a phase difference of $\pi$ (or $-\pi$) meaning:

$$\phi_{x,y} \in [-\pi, -\frac{\pi}{2}] \cup [-\frac{\pi}{2}, \pi].$$

If $\phi_{x,y} \in [\frac{\pi}{2}, \pi]$ then $x$ is leading, and the time series $y$ is leading if $\phi_{x,y} \in [-\pi, -\frac{\pi}{2}]$.

### 2.2.3 Significance level and background noise of the distribution

The statistical significance of wavelet power spectrum of the observed time series can be assessed relative to the null hypothesis that the signal generating the process is
stationary. Since the series in this study cannot be said to be stationary at level, stationarity is induced by log-differencing them as explained in Section 2.1. This transformation ensures that the observed time series is normal and can be modelled as a Gaussian AR(1) process. We assume that null hypothesis for power spectrum is normally distributed as AR(1) process. This assumption affects the acceptance of null hypothesis for the power spectrum of each time series or for the co-spectrum of any two time series as well as their coherence. The colour of the noise on the other hand is important for both the spectrum and the co-spectrum while the wavelet coherence is insensitive to the choice of the color. Figure 1 shows that red noise is an appropriate background to test against, the theoretical AR1 spectrum for the power decay closely matching the observed spectrum. In what follows, we choose to work with red noise process given that the observed data were log-transformed to induced stationarity, but also investigate the implication of red noise for the null hypothesis. The following simple AR(1) model will serve to illustrate the difference between white and red noise

\[ y_t = m + \alpha y_{t-1} + \varepsilon_t \]  

(7)
where \( y_0 = 0 \), \( m \) is a constant, \( \alpha \) is the autocorrelation coefficient and \( \varepsilon_t \sim N(0, \sigma^2) \). The white noise model is implied by setting \( m = 0 \) and \( \alpha = 0 \) (that is, \( y_t = \varepsilon_t \)) while the red noise results by setting \( m = 0 \) and \( \alpha \to 1 \). For the red noise, the Fourier power spectrum is given by

\[
P_k = \frac{1 - \alpha^2}{1 + \alpha^2 - 2\alpha \cos \left( \frac{2\pi k}{N} \right)}
\]  

(8)

where we see that \( P_k = 1 \) for white noise. Although Torrence and Compo (1998) have shown how the statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by an AR(0) or AR(1) stationary process with a certain background Fourier power spectrum, for more general processes one has to rely on Monte-Carlo simulations. Torrence and Compo (1998) computed the white noise and red noise wavelet power spectra, from which they derived, under the null, the corresponding distribution for the local wavelet power spectrum at each time \( m \) and scale \( s \) as follows:

\[
\frac{|W_x^m(s)|^2}{\sigma_X^2} \sim \frac{1}{2} P_k \chi_\nu^2
\]  

(9)

where \( \nu \) is equal to 1 for real and 2 for complex wavelets. According to Torrence and Compo (1998), if two time-series have theoretical Fourier spectra \( P^X_k \) and \( P^Y_k \) as defined in equation (8), and are both \( \chi^2 \) distributed then the cross-wavelet distribution is given by (Torrence and Compo, 1998, p. 76)

\[
\frac{|W^X_m(s)\tilde{W}^Y_m(s)|^2}{\sigma_X \sigma_Y} \sim \frac{Z(p)}{\nu} \sqrt{P^X_k P^Y_k}
\]  

(10)

where \( Z(p) \) is the confidence level associated with the probability \( p \) for a probability density function defined by the square root of the product of two \( \chi^2 \) distributions. In another context, Priestley (1981, p. 703) derives the asymptotic distribution of the estimated cross-amplitude power and shows that the asymptotic distribution depends on the coherence. In particular, he shows the variance of the estimated cross-amplitude power at frequency \( \omega \) is

\[
\text{var} \{\hat{\alpha}_{xy}(\omega)\} \sim \frac{C_{xy}(\omega)}{2N^2} \alpha_{xy}^2(\omega) \left\{ 1 + \frac{1}{|C_{xy}(\omega)|^2} \right\}
\]  

(11)

This result is an important demonstration of the relationship between the variability of the cross-amplitude estimate and the coherence of the series. It shows that at all frequencies where coherence is low, the estimate of the cross-amplitude may have an extremely large variance (Priestley, 1981, p. 703). We observe that this analogy may well be true of wavelet cross spectrum as well. Aside from this insight into the noted relationship, this conclusion has no damaging implication for the distribution in Equation (10) or for our results. For testing the statistical significance of results
we make use of Monte Carlo simulation approach. We specifically make use of ARMA(1,0) background noise to imitate the red noise. Again, we must mention that wavelet coherence is insensitive to the noise colour and the choice of background colour may not affect the result reported for coherence.

3 Results and discussion

In this section we present the results based on the wavelet analysis. Noteworthy to mention that in Figures 2 to 4, observation 100 corresponds to 1968M5, 200 corresponds to 1976M9, 300 corresponds to 1985M1, 400 corresponds to 1993M5 and 500 corresponds to 2001M9. It is evident from Fig. (top) that the common features in the wavelet high power of nominal stock returns and inflation are evident between 1985 and 1993, at $8 \sim 16$ months and $16 \sim 32$ months time scales. However, the significant common island is evident only between 1985-1993 period at approximate time scale of $8 \sim 16$ months. Not many differences are seen when same relationship is analyzed using real stock returns. However, we do not observe any clear pattern in these periods and, hence, nothing can be said with precision, whether this is merely a coincidence or real correlation.

We, therefore, analyze the data through cross wavelet power and the results are presented in Fig. (top). We observe that, at all frequencies corresponding to high power, inflation and nominal stock returns are out of phase at all significant common islands.

It is, however, noteworthy to mention that wavelet cross-spectrum (i.e., cross wavelet) describes the common power of two processes without normalization to the single wavelet power spectrum. This can produce misleading results, because one essentially multiplies the continuous wavelet transform of two time series. For example, if one of the spectra is locally and the other exhibits strong peaks, then, peaks in the cross-spectrum can be generated having no relation with the two series. This leads us to conclude that wavelet cross spectrum is not suitable to test the significance of relationship between two time series. We, hence, rely on the wavelet coherency as it is able to detect true inter-relation between two time series. The wavelet coherency is used to identify both frequency bands and time intervals within pair of time series show the co-movement. The results of cross-wavelet coherency for inflation and nominal stock returns are shown in Fig. (top). Our results based on cross-wavelet coherency indicate arrows mostly pointing upward left at $8 \sim 16$ months time scale (frequency) over different periods. This implies that inflation and nominal stock returns are out of phase (anti-cyclical) for most of the periods. Similarly, at more than 128 months time scale over 1975-1993 time period, inflation and stock prices are out of phase (negative relationship). However, there are some short periods like 1991-1993 over $16 \sim 32$ months and 1965-1970 (for real stock returns) over $32 \sim 64$ months scale, where stock returns and inflation are in phase or cyclical (positive relation).

We, therefore, confirm that fisher effect was evident for a very limited period of time.
Figure 2: The continuous wavelet power spectrum of both nominal stock returns and inflation (top) and real stock returns and inflation (bottom)

Note: The thick black contour designates the 5% significance level against red noise and the cone of influence (COI) where edge effects might distort the picture is shown as a lighter shade. The color code for power ranges from white (low power) to dark gray (high power). Y-axis measures frequencies or scale-in months and X-axis represent the time period studied.

(1965-1970) over 32 ~ 64 months time scale and (1991 to 1993) over 16 ~ 32 months time scale during the studied sample period.
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Figure 3: Cross wavelet power spectrum of the nominal stock returns and Inflation (top) and real stock returns and Inflation (bottom)

Note: The thick black contour designates the 5% significance level against red noise which estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from white (low power) to dark gray (high power). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with stock return is lagging. To the right and down, with stock return is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with stock return is leading. To the left and down, with stock return is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having anti-cyclical effect on each other.

Next, following Kim and In (2005) we test whether nominal stock returns, compared
Figure 4: Cross wavelet coherency of the nominal stock returns and Inflation (top) and real stock returns and Inflation (bottom)

Note: The thick black contour designates the 5% significance level against red noise which estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from white (low power) to dark gray (high power). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with stock return is lagging. To the right and down, with stock return is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with stock return is leading. To the left and down, with stock return is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having anti-cyclical effect on each other.

to the real returns, correspond differently to the inflation over the different time horizons. Our results, based on real stock returns are similar to the results based on nominal stock returns at least for wavelet power spectrum (Figure 2(bottom)) and
cross wavelet power spectrum (Figure 3 bottom). However, results based on wavelet coherency show that real stock returns and inflation are out of phase at 32 ∼ 64 months time scale over 1965-1970 time period. Results based on nominal and real stocks returns are, therefore, not robust at 32 ∼ 64 months time scale over 1965-1970. Overall, the results indicate that stock returns have not played any significant role as an inflation hedge during the studied sample period, except for short period of 1991-1993 over 16 ∼ 32 months time scale. This set of result is robust to the use of both real as well as nominal stock returns. Our results are institutive and appealing, in the sense that, the relation between stock prices and inflation has been explored across frequencies as well as over time. These results, nevertheless, could not have been arrived at utilizing time domain analysis based on conventional econometric techniques.

4 Conclusions

In this study, we investigated the stock price-Inflation nexus using new approach based on continuous wavelet transform to study the Fisher hypothesis. To test the relationship, we adopted wavelet power spectrum, cross-wavelet power spectrum and cross-wavelet coherency. Our results indicated that, inflation and stock returns (real and nominal) were out of phase (anti-cyclical or negative) for most of the time periods and frequencies whereas, stock returns (both nominal/real) and inflation were in phase (positive relation) for very short period of time over 16 ∼ 32 month frequency. Our results, therefore, suggested that for the entire sample period, stock returns had not played any role as an inflation hedge, except for short period of 1991-1993 over 16 ∼ 32 months frequency scale. With this methodology, we were, therefore, able to unravel time and frequency dependent relationship between inflation and stock returns, which could have been impossible to explore with conventional econometric techniques.

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References


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